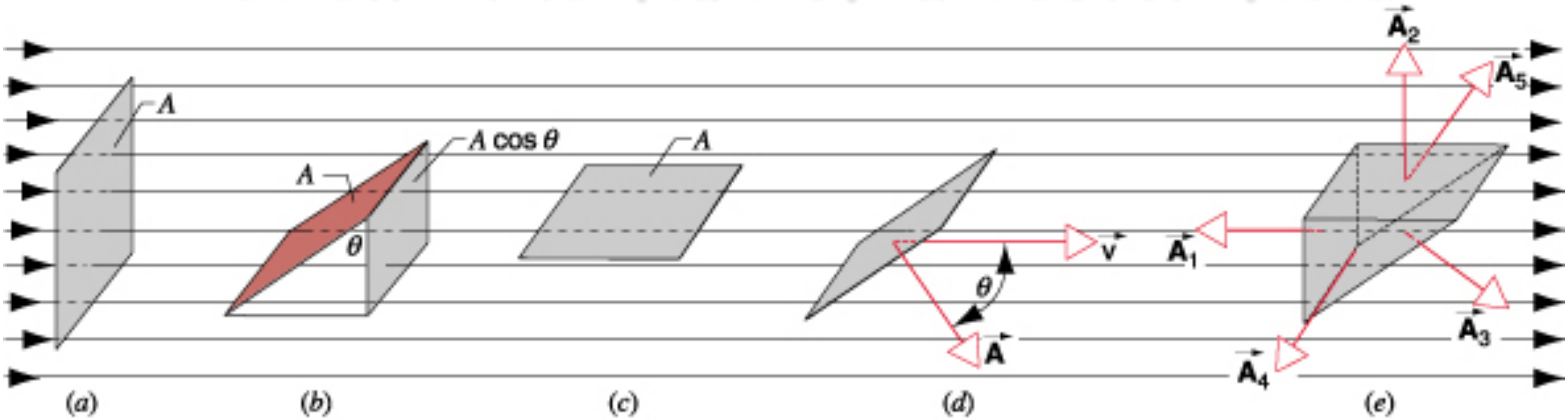
Chapter 21: Gauss' law
Tuesday September 13th

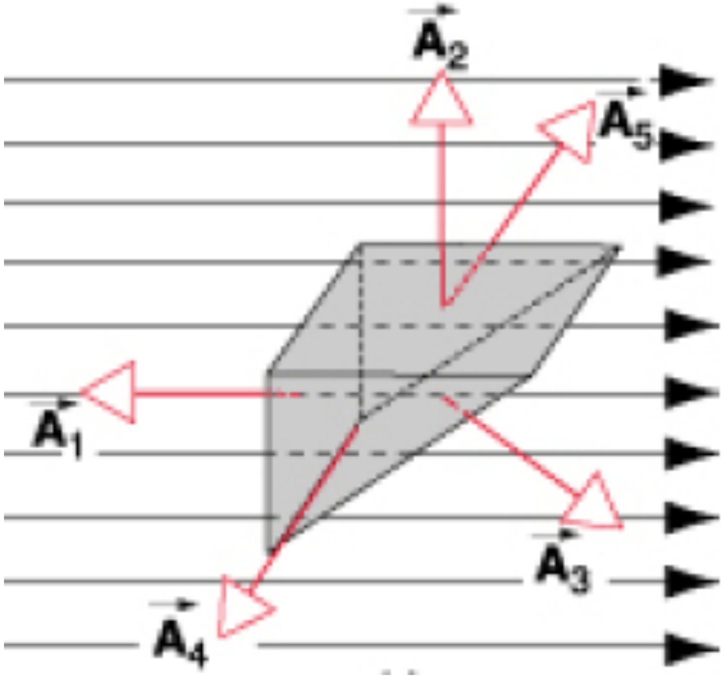
LABS START THIS WEEK

- Quick review of Gauss' law
  - The flux of a vector field
  - The shell theorem
- Gauss' law for other symmetries
  - A uniformly charged sheet
  - A uniformly charged cylinder
- Gauss' law and conductors
- Electrostatic potential energy (more likely on Thu.)

Reading: up to page 363 in the text book (end Ch. 21)

Review: the flux of a vector field





What if there are multiple surface elements to consider?

Then,

Sign determined

$$\Phi = \sum \vec{\mathbf{v}} \cdot \vec{\mathbf{A}}.$$

For a closed surface, we ALWAYS choose $\vec{\mathbf{A}}$ to point outwards. This is very important for Gauss' Law!!

The flux of an electric field

Gauss' law is concerned with the flux of E through closed surfaces

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

- **You may recall that when we developed our graphical representation of electric field lines, the electric field strength was proportional to the number of field lines crossing a unit area perpendicular to the field.**
- **Consequently,**

$$Flux = \sum \frac{\#\,of\ \ field\ lines}{\perp area} \times (\perp area) = \sum \#\,of\ \ field\ lines.$$

- **In other words, the flux of E through a surface is proportional to the number of field lines penetrating the surface.**
- **This is the essence of Gauss' law.**
- **Recall also that the number of field lines is related to the number of charges producing the electric field.**

The flux of an electric field

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$$\boxed{\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_o}}$$

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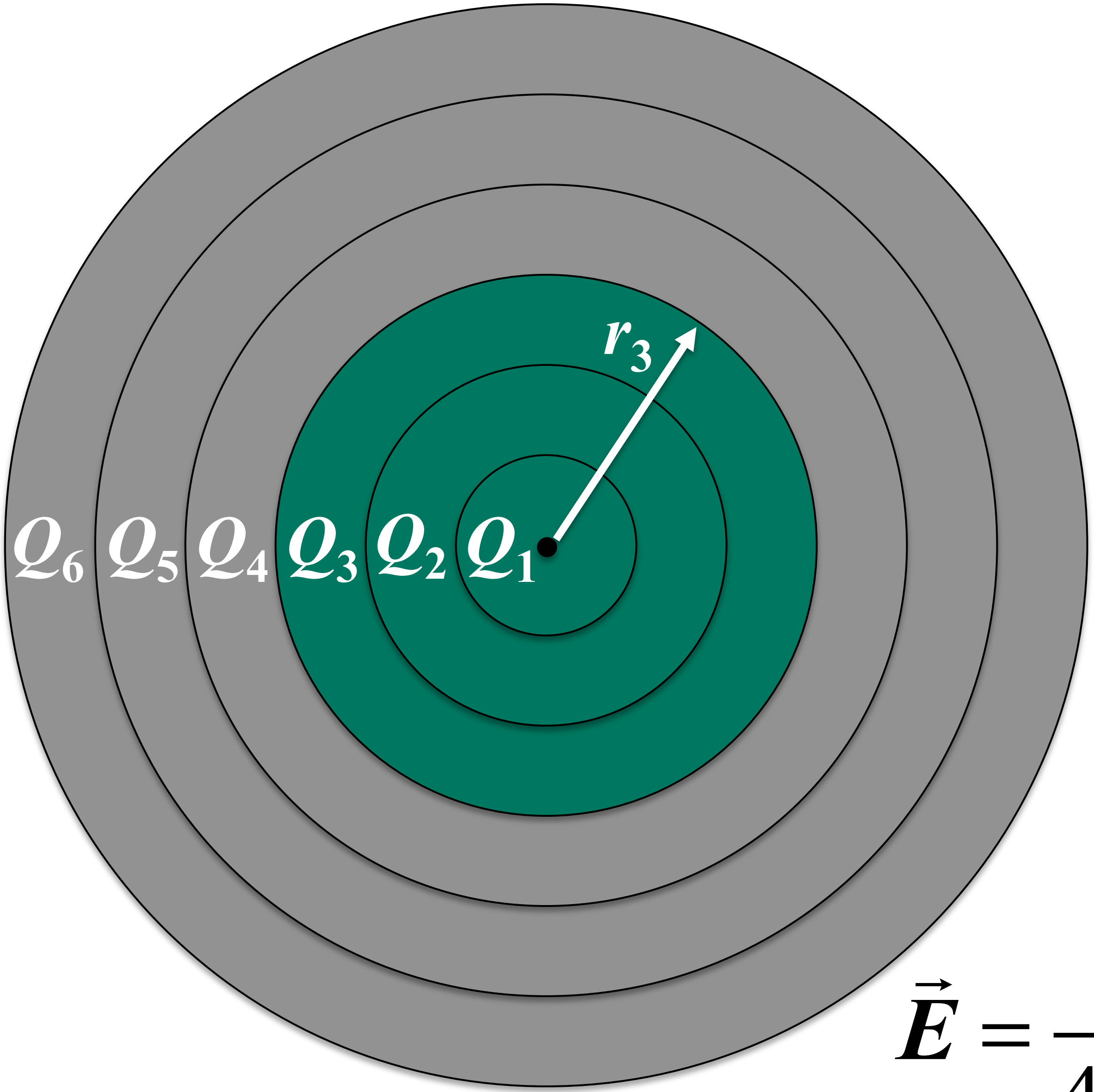
Gauss' law and the shell theorem

Key ideas:

- **Symmetry is crucial – symmetries that work:**
  - **Spherical (solid sphere, spherical shell, etc..)**
  - **Cylindrical (line charge, tube of charge, etc..)** **Today**
  - **Planar (sheet of charge, slab of charge, etc..)**
- **Construct an imaginary (Gaussian) surface to aid in calculating the field; you then calculate the flux through this surface.**
- **For the spherical case, the Gaussian surface must be spherical and concentric with the charge, otherwise the surface integral is undetermined (similar principles apply to the other symmetries).**
- **The flux through the Gaussian surface and, therefore, the field, depends <u>only</u> on the charge inside the surface;**
  - **The electric field at radius $r = R$ knows nothing about charge at larger radii, $r > R$;**
  - **If all charge is contained within a radius R then, for radii $r > R$, it appears as though all of the charge is located at the center of the sphere, i.e., field knows nothing about charge distribution.**

Gauss' law and the shell theorem



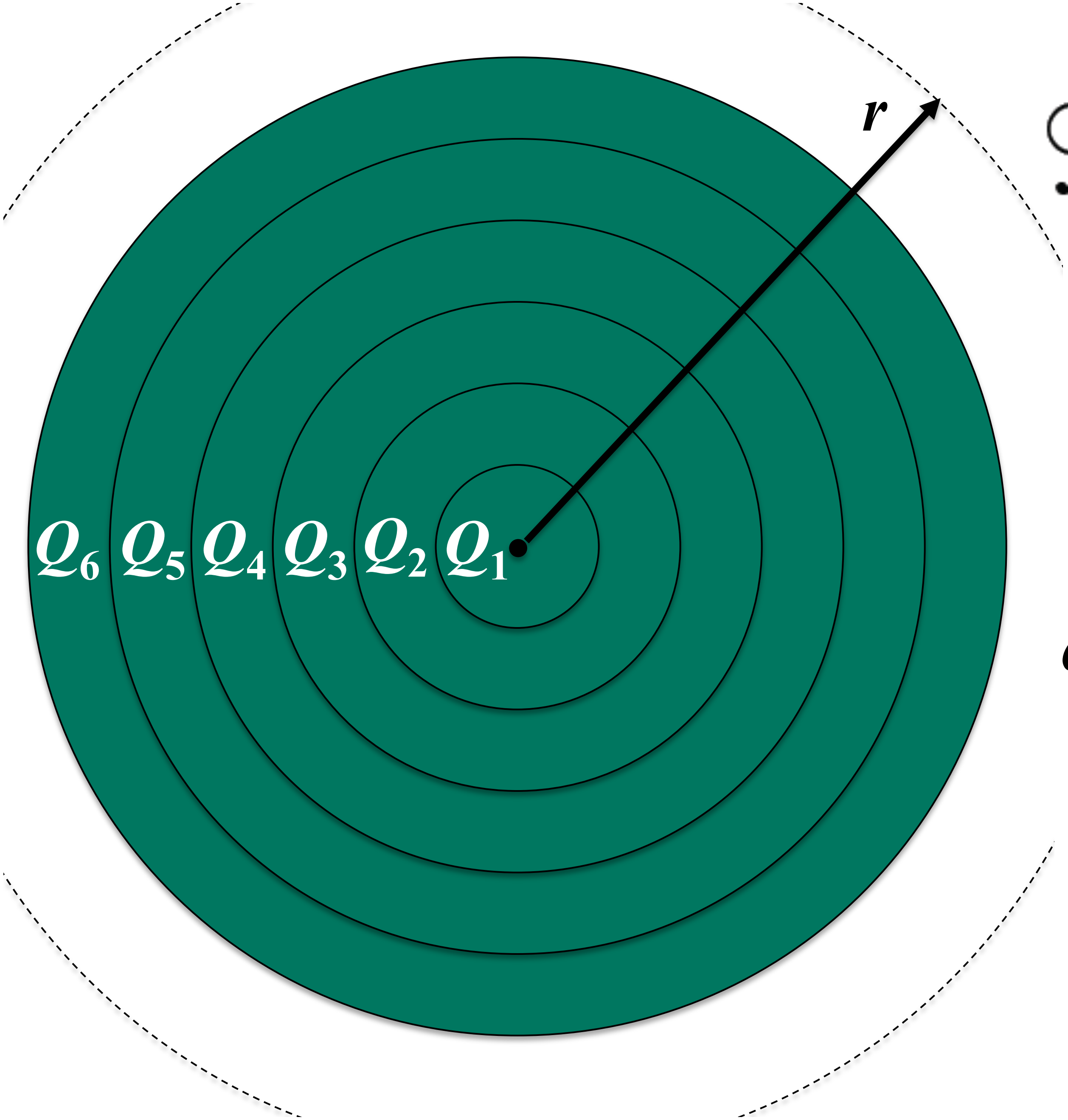
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_o}$$

$$4\pi r^2 E = \frac{q_{enc}}{\varepsilon_o}$$

$$q_{enc} = Q_1 + Q_2 + Q_3$$

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_o} \frac{Q_1 + Q_2 + Q_3}{r_3^2} \hat{\boldsymbol{r}}$$

Gauss' law and the shell theorem



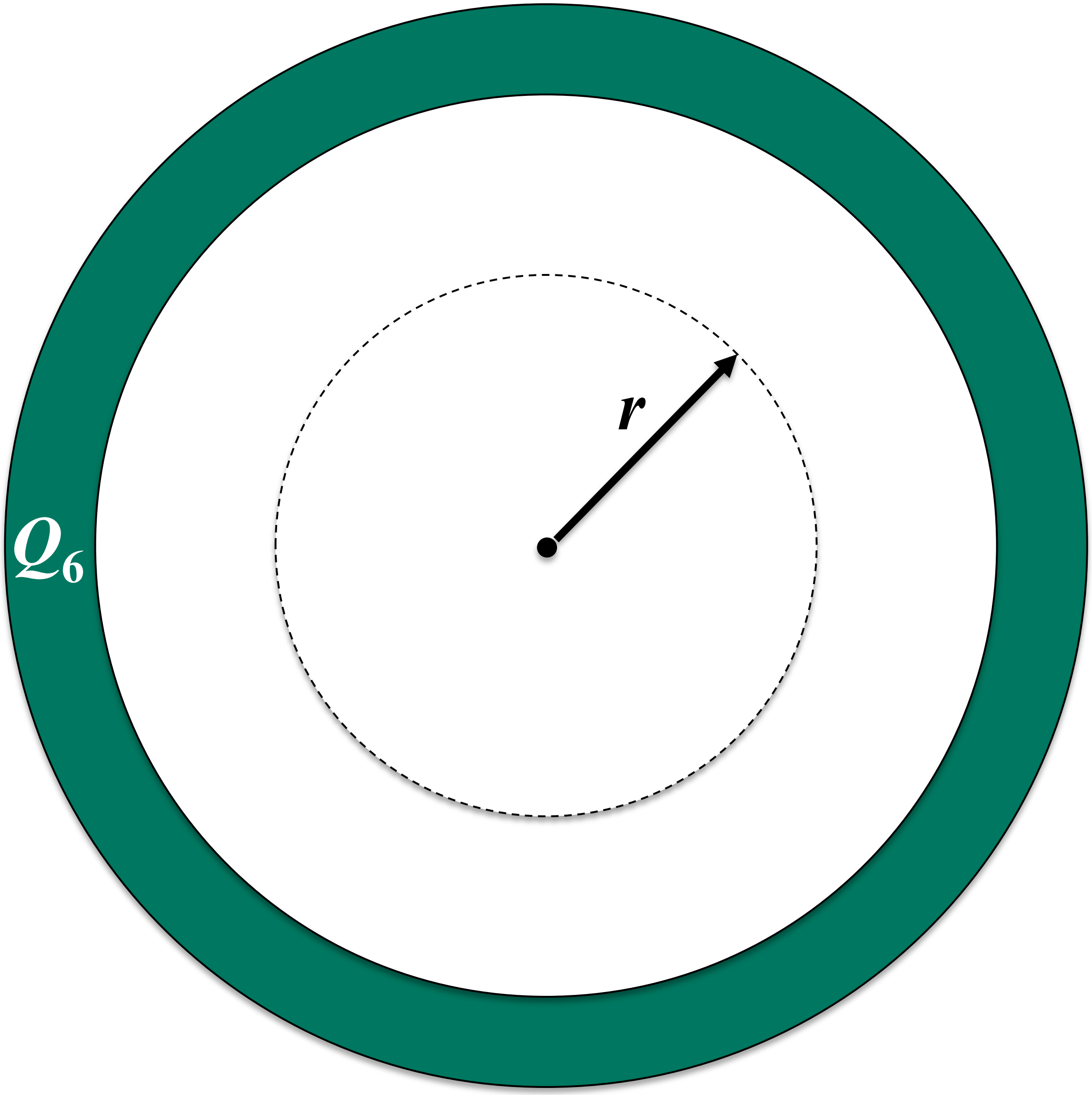
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_o}$$

$$4\pi r^2 E = \frac{q_{enc}}{\varepsilon_o}$$

$$q_{enc} = \sum_{i=1}^{6} Q_i = Q_{tot}$$

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_o} \frac{Q_{tot}}{r^2} \hat{\boldsymbol{r}}$$

Gauss' law and the shell theorem



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_o}$$

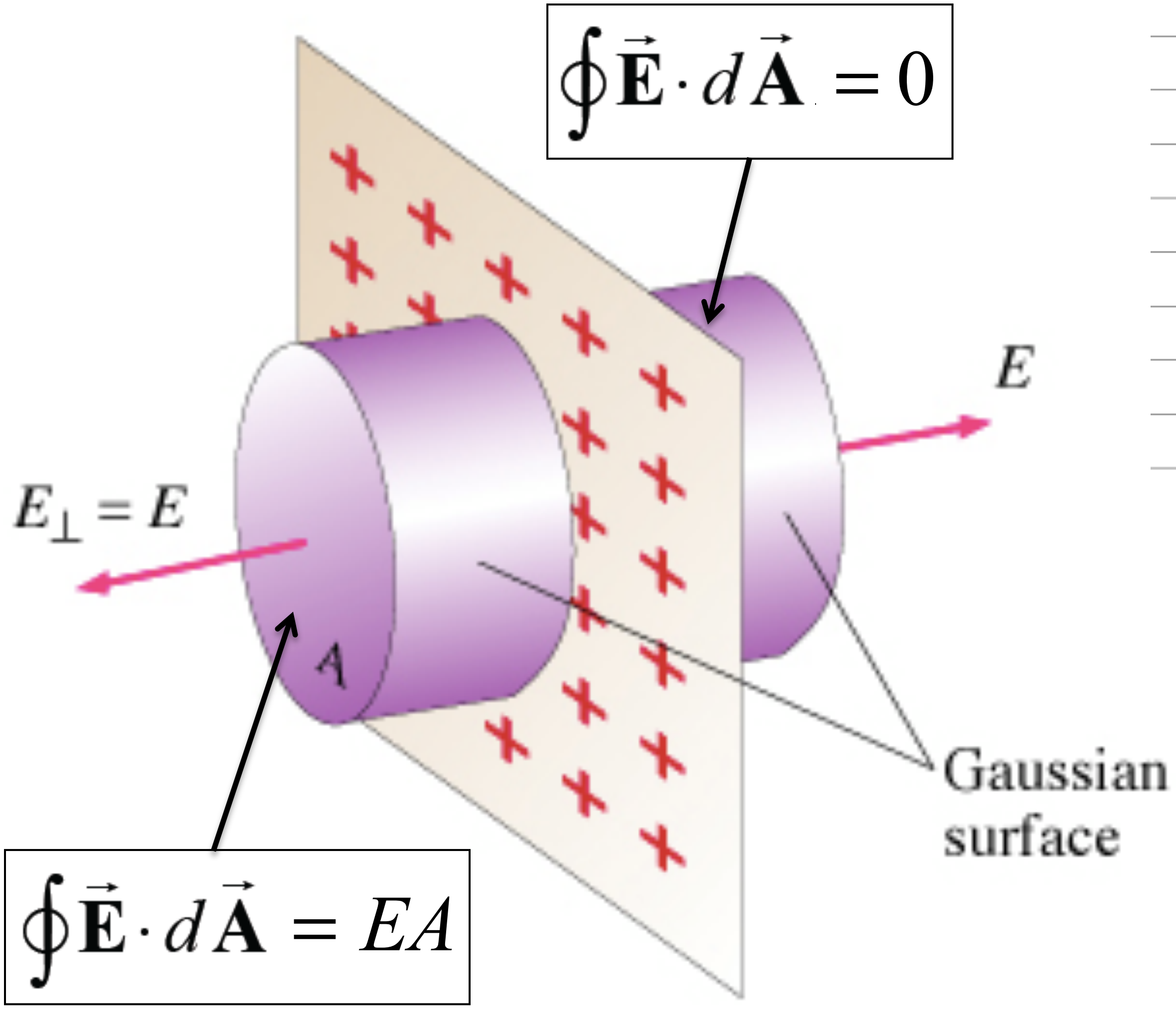
$$4\pi r^2 E = \frac{q_{enc}}{\varepsilon_o}$$

$$q_{enc} = 0$$

$$\vec{E} = 0$$

Gauss' law for planar symmetry (sheet charge)

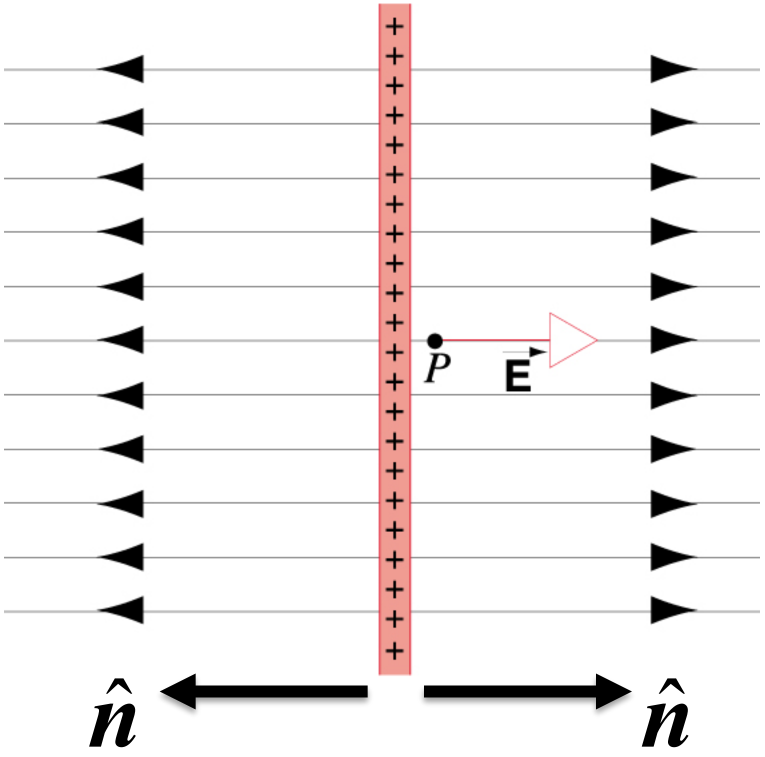
- Uniform field (does not depend on position)
- Everywhere perpendicular to the surface
- Surface charge density, σ; units are C/m^2



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

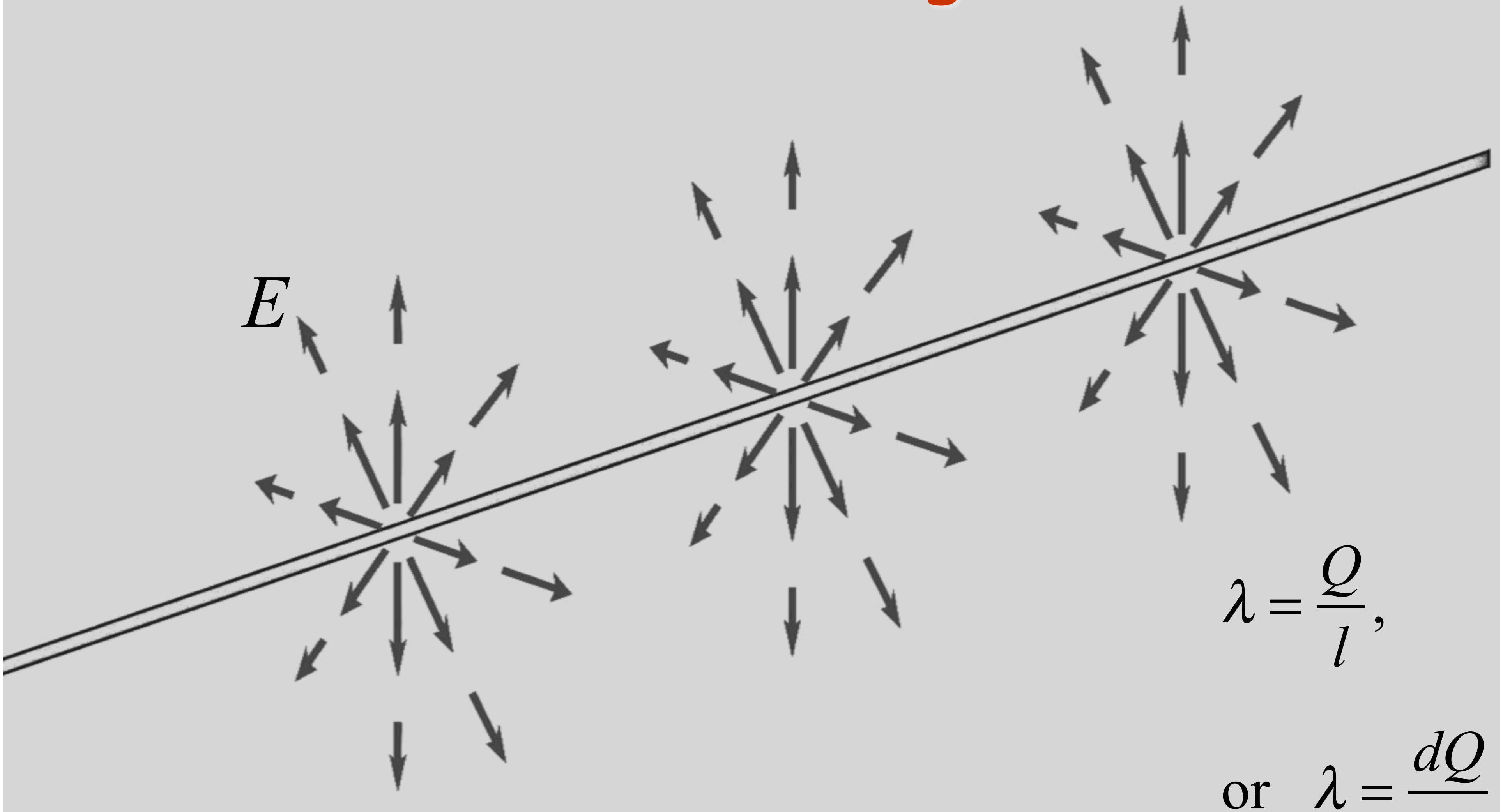
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA$$

Side view



$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{n}$$

Line of Charge

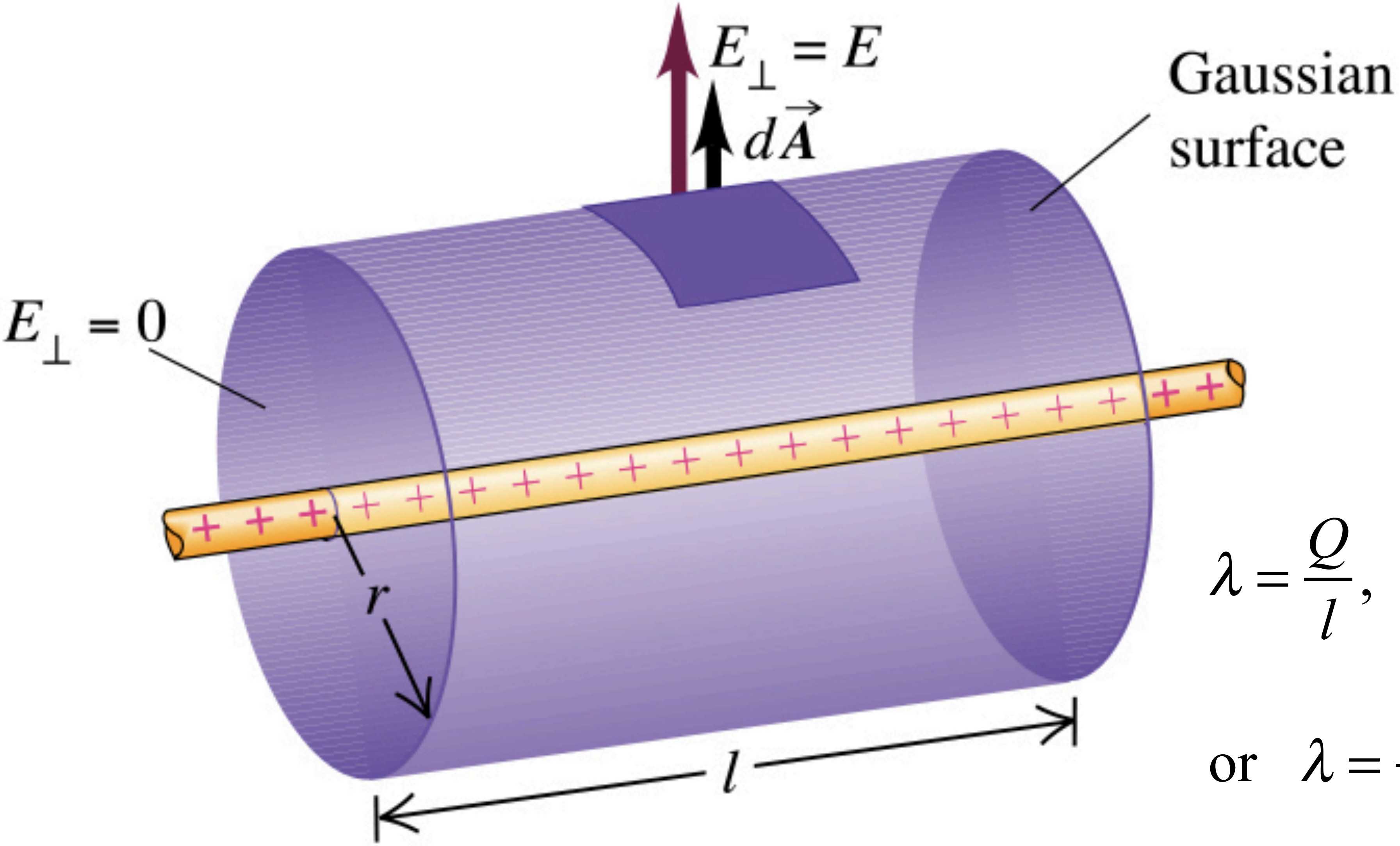


$$\lambda = \frac{Q}{l},$$

$$\text{or} \quad \lambda = \frac{dQ}{dl}$$

Line charge density, or charge per unit length, λ, in Coulombs per meter.

Line of Charge



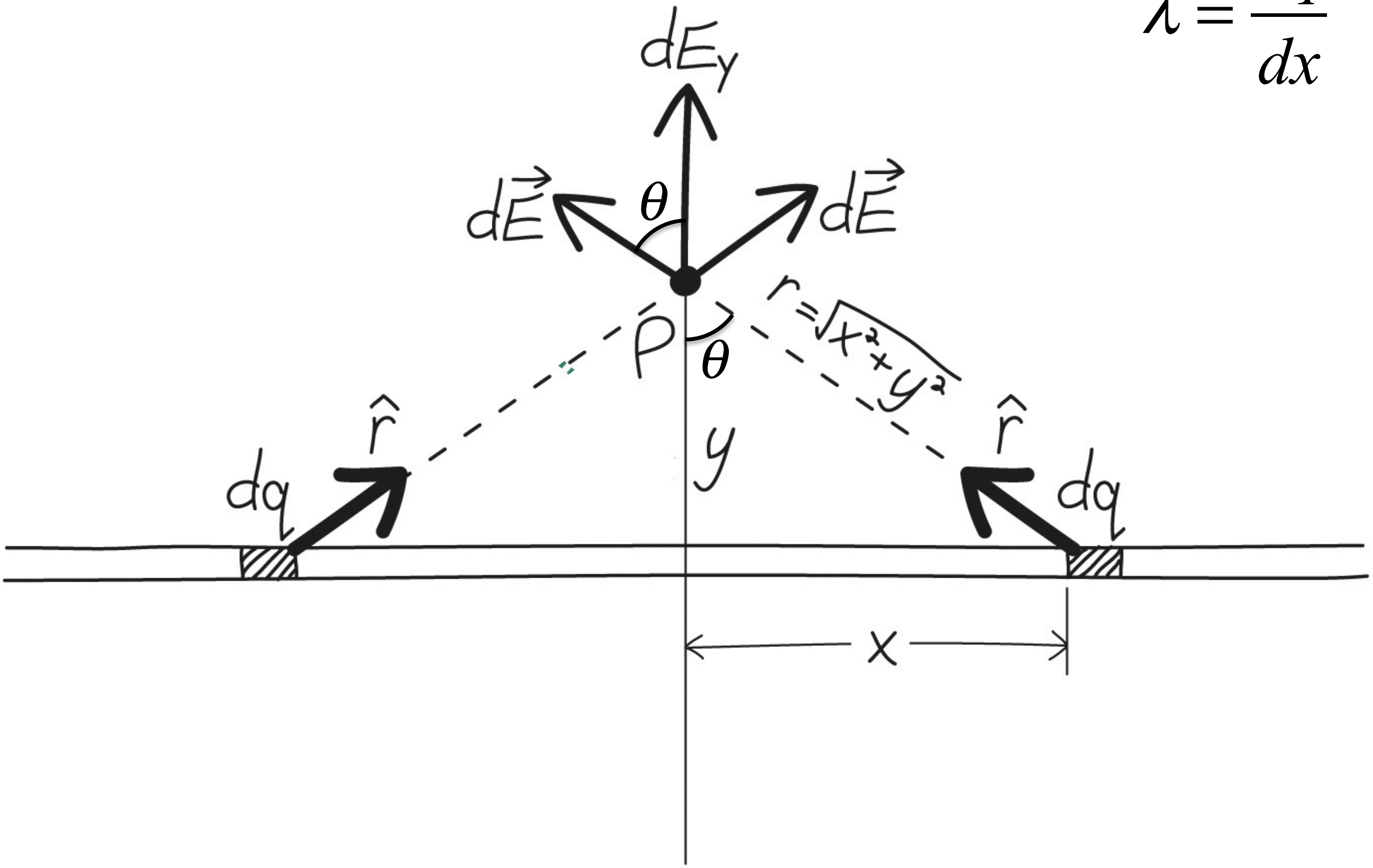
$$\lambda = \frac{Q}{l},$$

$$\text{or} \quad \lambda = \frac{dQ}{dl}$$

Line charge density, or charge per unit length, λ, in Coulombs per meter.

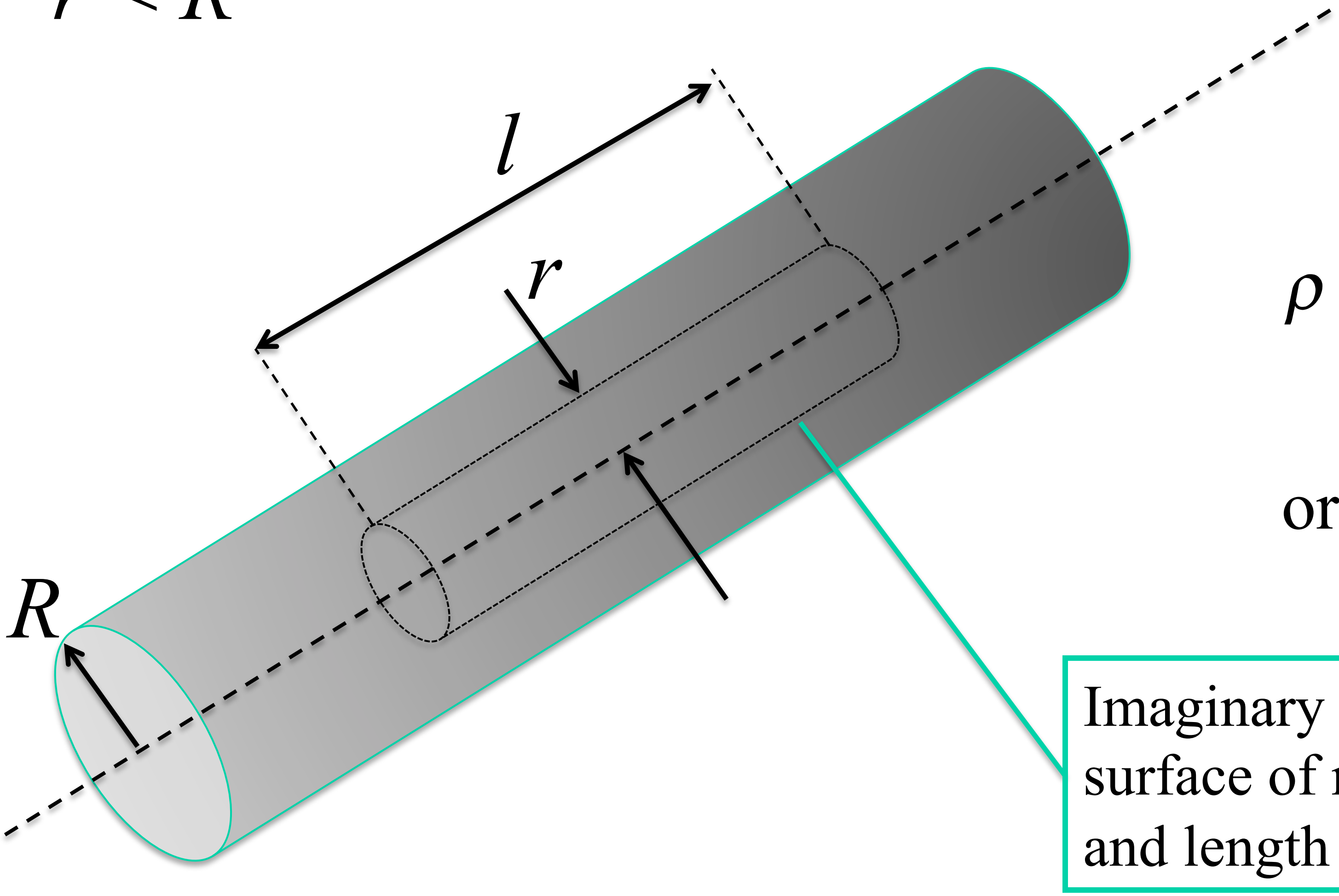
Recall the horrible integration

$$\lambda = \frac{dq}{dx}$$



Charged cylinder with finite radius *R*

$r < R$



Imaginary Gaussian surface of radius r and length l.

$$\rho = \frac{Q}{V},$$

or $$\rho = \frac{dQ}{dV}$$

Volume charge density, or charge per unit volume, ρ, in Coulombs per m^3.

Charge densities

In 1D (a line or wire):

$$\lambda = \frac{Q}{L}, \qquad \text{or} \qquad \lambda = \frac{dQ}{dL}$$

λ is the line charge density, or charge per unit length, in Coulombs per meter. L represents length, and Q is charge.

In 2D (a surface or sheet):

$$\sigma = \frac{Q}{A}, \qquad \text{or} \qquad \sigma = \frac{dQ}{dA}$$

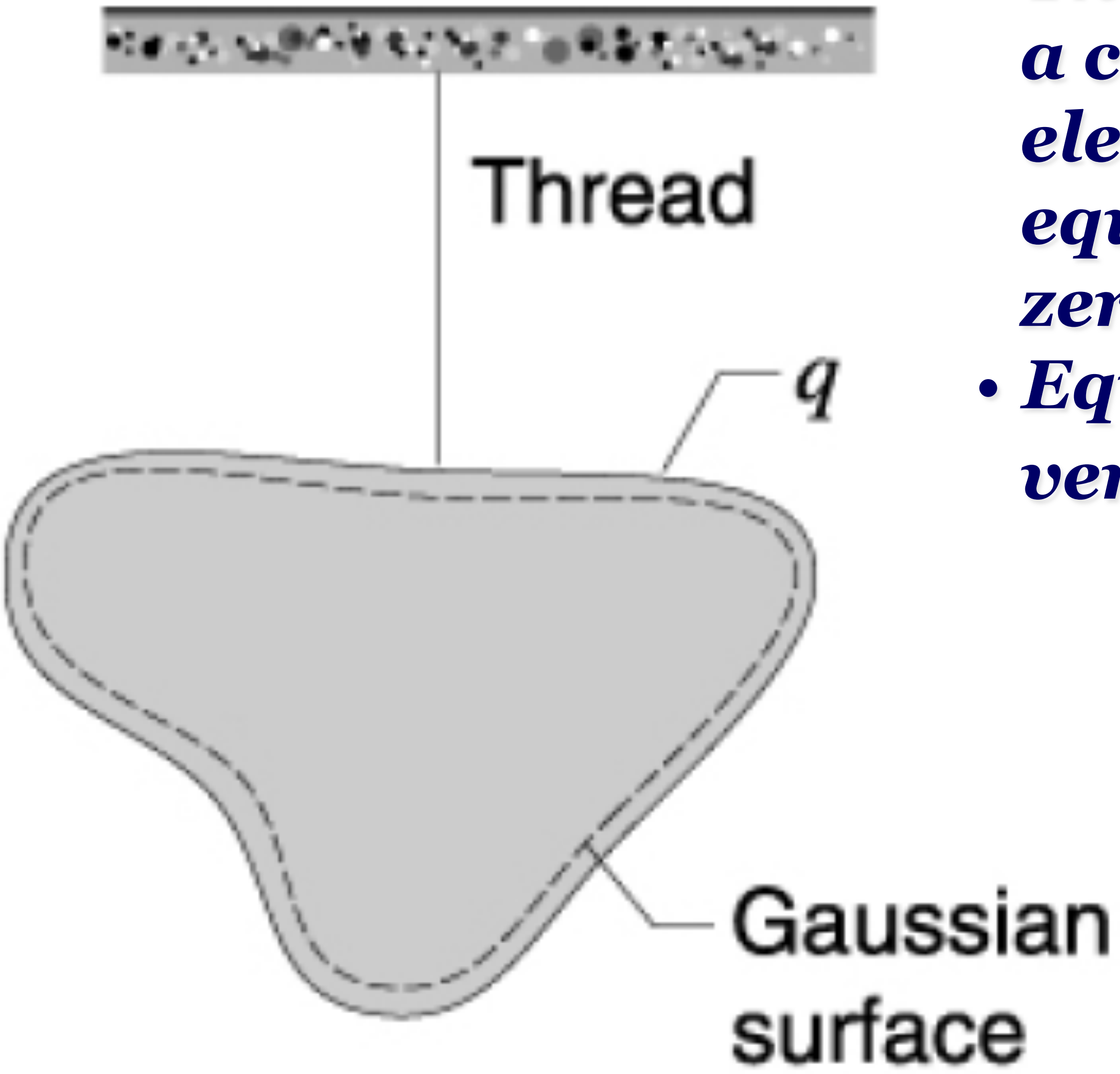
σ is the surface charge density, or charge per unit area in Coulombs per meter^2; A represents area, and Q is charge.

In 3D (a solid object):

$$\rho = \frac{Q}{V}, \qquad \text{or} \qquad \rho = \frac{dQ}{dV}$$

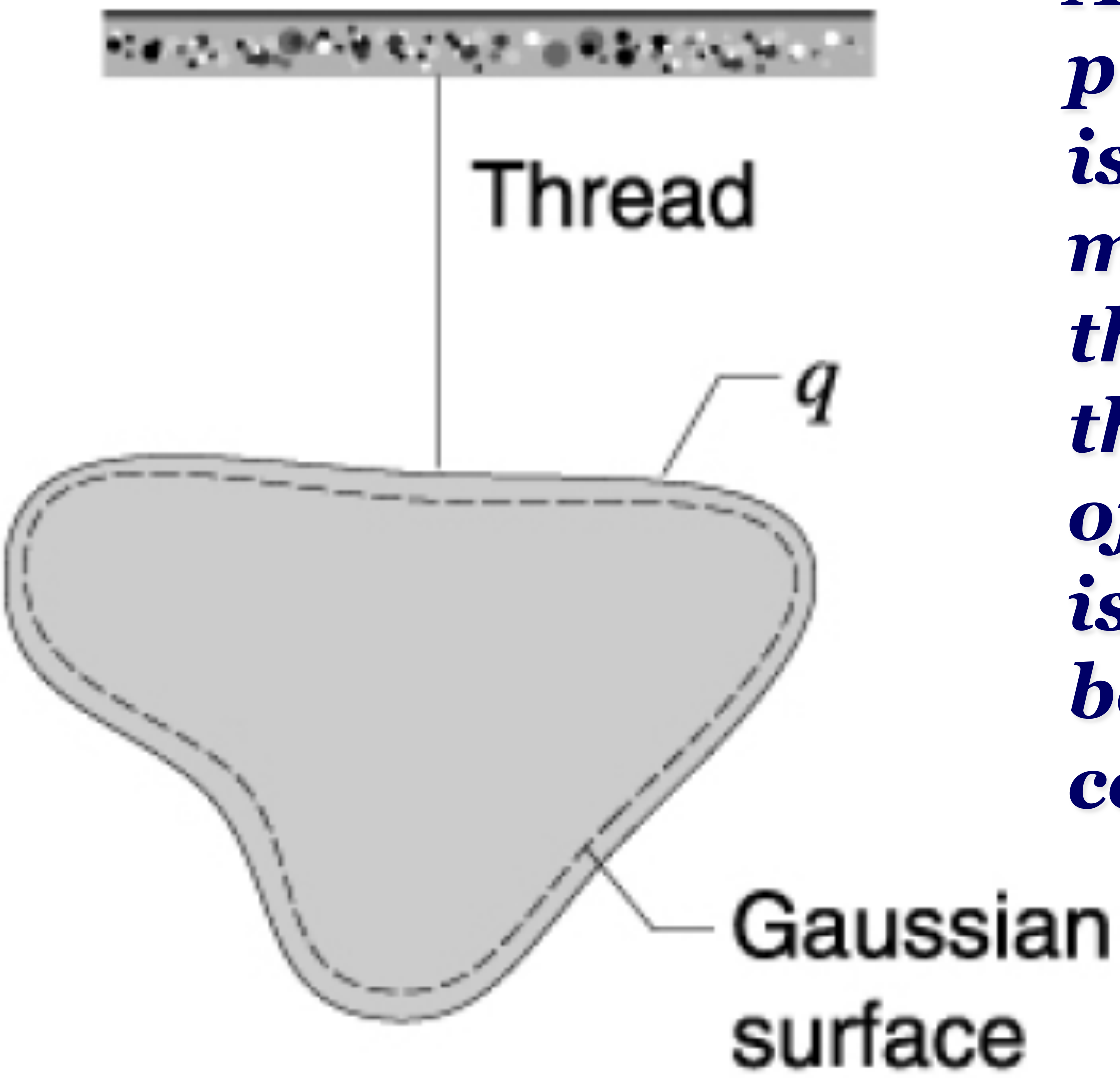
ρ is the volume charge density, or charge per unit volume in Coulombs per meter^3. V represents volume, and Q is charge.

Gauss’ law and conductors



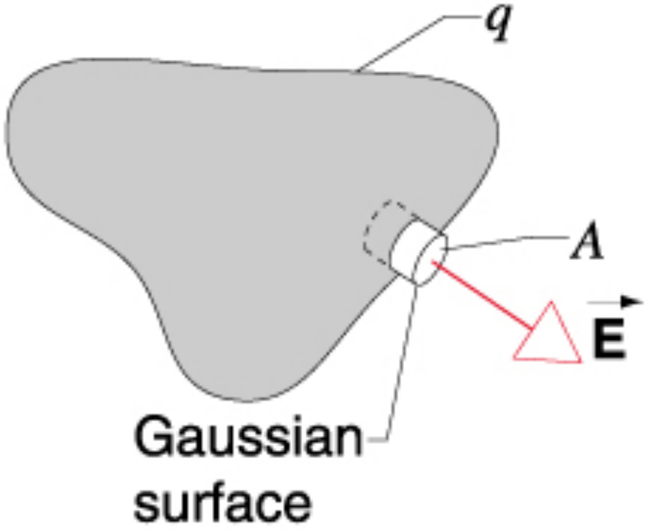
- ***The electric field inside a conductor which is in electrostatic equilibrium must be zero.***
- ***Equilibrium is reached very quickly** **($<10^{-9}$ s).***

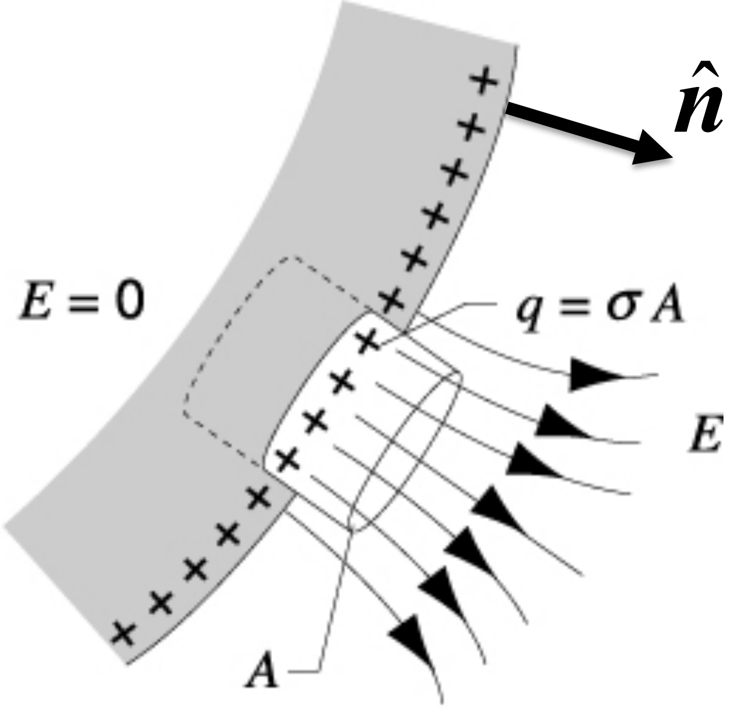
Gauss' law and conductors



An excess charge placed on an isolated conductor moves entirely to the outer surface of the conductor. None of the excess charge is found within the body of the conductor.

The electric field outside a conductor





$$\vec{E}_{@S} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

$$\Phi_E = \oint_{surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$= \oint_{outer\ cap} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \cancelto{0}{\oint_{inner\ cap} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}} + \cancelto{0}{\oint_{side\ walls} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}}$$

Conductors necessarily distort field lines